Imprecise-probabilistic processes - Part II

from a financial mathematical perspective

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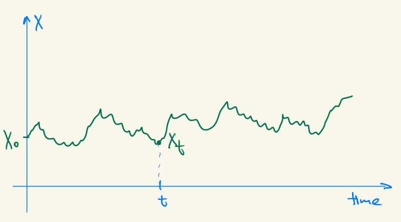
Pricing in Financial Markets



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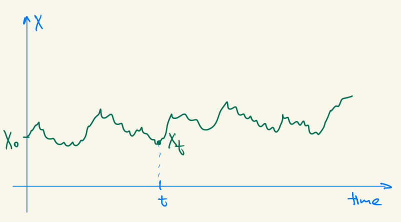

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Examples:

- **Bachelier model:**

$$X_t = X_0 + mt + \sigma B_t,$$

where $X_0 \in \mathbb{R}$ is the current state, $m \in \mathbb{R}$ is the drift parameter, $\sigma > 0$ is the volatility and $(B_t)_{t\geq 0}$ is a Brownian motion.

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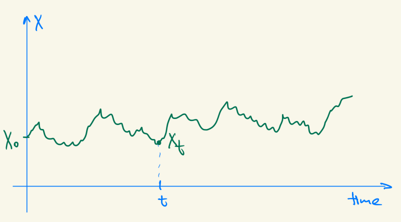

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  - **Black Scholes model:**

    $$X_t = X_0 \exp\left(\sigma B_t + (m - \tfrac{1}{2}\sigma^2)t\right),$$

    where $X_0 > 0$ is the current state, $m \in \mathbb{R}$ is the drift parameter, $\sigma > 0$ is the volatility and $(B_t)_{t \geq 0}$ is a Brownian motion.

Pricing in Financial Markets

- Goal: Pricing of a **contingent claim**

$$H = f(X_T)$$

with maturity $T > 0$ and payoff function $f\colon \mathbb{R} \to \mathbb{R}_+$.

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- The seller is interested in the **hedging problem**

$$\pi + \int_0^T \vartheta_s \, dX_s = H,$$

where $\pi \in \mathbb{R}_+$ is the fair price and $(\vartheta_s)_{s \in [0,T]}$ is a replicating portfolio. Here, the stochastic integral $\int_0^T \vartheta_s \, dX_s$ describes the gains/losses from dynamic trading in the time-interval $[0, T]$.

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- The hedging problem has often no solutions and can be relaxed to the **super-hedging problem**

$$\pi + \int_0^T \vartheta_s \, dX_s \geq H,$$

where $\pi \in \mathbb{R}_+$ is a super-heging price and $(\vartheta_s)_{s\in[0,T]}$ is a super-replicating portfolio.

Pricing in Financial Markets

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- Let $u\colon [0, T] \times \mathbb{R} \to \mathbb{R}$ be the solution of the *boundary value problem*

$$\begin{cases} \partial_t u + \frac{1}{2}\sigma^2 \partial_{xx} u = 0 & \text{on } [0, T) \times \mathbb{R} \\ u(T, \cdot) = f & \text{on } \mathbb{R}. \end{cases}$$

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- Under reasonable assumption, its solution u is $C^{1,2}$, and we obtain from Ito's lemma that

$$\begin{aligned} H &= f(X_T) = u(T, X_T) \\ &= u(0, X_0) + \int_0^T \partial_x u(s, X_s)\, dX_s + \int_0^T \partial_t u(s, X_s) + \frac{1}{2}\sigma^2 \partial_{xx} u(s, X_s)\, ds \\ &= u(0, X_0) + \int_0^T \partial_x u(s, X_s)\, dX_s. \end{aligned}$$

Hence, $u(0, X_0)$ is the **fair price** and $\partial_x u(s, X_s)$ is the **replicating strategy**.

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- A **semigroup** on $\mathrm{C_b}$ is a family $(S(t))_{t\geq 0}$ of operators $S(t)\colon \mathrm{C_b} \to \mathrm{C_b}$ such that
  - $S(0)f = f$,
  - $S(t+s)f = S(t)\big(S(s)f\big)$ for all $s, t \geq 0$,
  - some sort of continuity.

  Here, $\mathrm{C_b}$ denotes the space of all bounded continuous functions $f\colon \mathbb{R} \to \mathbb{R}$.

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Examples:

- The **heat semigroup** is given by

$$\big(S(t)f\big)(x) := E\big[f(x + B_t)\big],$$

where $(B_t)_{t\geq 0}$ is a Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$.

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- More generally, for a **Markov process** $(X_t^x)_{t\geq 0}$ starting at $X_0^x = x$, the corresponding **transition semigroup** is defiend by $\big(S(t)f\big)(x) := E\big[f(X_t^x)\big]$.

Pricing in Financial Markets

- The **semigroup property** of the heat semigroup

$$\big(S(t)f\big)(x) := E\big[f(x+B_t)\big]$$

follows from the tower property of the conditional expectation and the properties of the Brownian motion. Indeed,

$$\begin{aligned}\big(S(t+s)f\big)(x) &= E\big[f(x+B_{t+s})\big]\\ &= E\Big[E\big[f(x+B_t+B_{t+s}-B_t)\big|\mathcal{F}_t\big]\Big]\\ &= E\Big[\tilde{E}\big[f(x+B_t+\tilde{B}_s)\big]\Big]\\ &= E\Big[\big(S(s)f\big)(x+B_t)\Big] = \Big(S(t)\big(S(s)f\big)\Big)(x),\end{aligned}$$

where $\tilde{B}_s \overset{d}{=} B_{t+s}-B_t \sim \mathcal{N}(0,s)$.

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- The solution of the *boundary value problem*

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is given by

$$u(t, x) = \big(S(\tau) f\big)(x)$$

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- In particular, the fair price π in the Bachelier model is given by

$$\pi = u(0, X_0) = \big(S(\sigma^2 T) f\big)(X_0) = E\big[f(X_0 + \sigma B_T)\big].$$

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- The fair price strongly depends on the model parameter σ (volatility).

Pricing in Financial Markets

- More generally, in actuarial science or financial mathematics, we are interested in **expected values** of the type

$$E[f(X_t^x)].$$

Examples:

  - **Fair value** of the option f written on the underlying (X_t^x) depending on time to maturity t and the today's state x.
  - **Expected loss** of the random factor X_t^x w.r.t. a loss function f.

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$$E[f(X_t^x)] = \int_{\mathbb{R}} f(y)\,\mu(dy) = \big(S(t)f\big)(x).$$

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- In order to compute the expectation, we need to know the distribution μ of (X_t^x), or the transition semigroup $(S(t))_{t\geq 0}$ of the Markov process $(X_t^x)_{t\geq 0}$.

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- However, in most situations, it is **impossible to identify** the precise probability distribution or transition semigroup (model uncertainty might appear due to insufficient data to perform reliable statistical estimations).

Imprecise Markov processes

Markov processes

- A **Markov process** on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$ is an adapted stochastic process $(X_t)_{t\geq 0}$ with

$$P\big(X_t \in A \mid \mathcal{F}_s\big) = P\big(X_t \in A \mid X_s\big)$$

for all $s \leq t$ and Borel sets $A \subset \mathbb{R}$.

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- The **transition semigroup** $(S(t))_{t\geq 0}$ of a Markov process is given by

$$\big(S(t)f\big)(x) := E\big[f(X_t) \mid X_0 = x\big] \quad \text{for all } f \in \mathrm{C_b} \text{ and } x \in \mathbb{R}$$

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(note that $S(0)f = f$).

- The local behaviour of the transition semigroup is given by the **generator**

$$Af = \lim_{h \downarrow 0} \frac{S(h)f - f}{h}$$

(for those functions for which the limit exists in a reasonable sense).

Markov processes

- Under reasonable assumptions there is a one-to-one realtion between

Markov process		**transition semigroup**		**generator**
	$\leftrightarrow$		$\leftrightarrow$	
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- In particular, the solution of the Kolmogorov equation

$$\begin{cases} \partial_t u = Au & \text{on } [0,T)\times\mathbb{R} \\ u(0,\cdot) = f & \text{on } \mathbb{R} \end{cases}$$

satisfies

$$E\big[f(X_t) \mid X_0 = x\big] = \big(S(t)f\big)(x) = u(t,x).$$

Markov processes

- Examples:

1) **Brownian motion** $\leftrightarrow$ **heat semigroup** $\leftrightarrow$ **generator**

$$(B_t)_{t\geq 0} \quad \leftrightarrow \quad (S(t)f)(x) = E\big[f(x+B_t)\big] \quad \leftrightarrow \quad Af = \tfrac{1}{2}f''$$

The solution of the Kolmogorov equation (heat equation)

$$\begin{cases} \partial_t u = \frac{1}{2}\partial_{xx} u & \text{on } [0,T)\times\mathbb{R} \\ u(0,\cdot) = f & \text{on } \mathbb{R} \end{cases}$$

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2) **Markov process modelled as SDE** $\leftrightarrow$ **transiton semigroup**

$dX_t^x = \mu(X_t^x)dt + \sigma(X_t^x)dB_t$ with $X_0^x = x$ $\quad\quad$ $(S(t)f)(x) = E\big[f(X_t^x)\big]$

$\updownarrow$

generator: $Af = \mu f' + \frac{1}{2}\sigma^2 f''$

Imprecise Markov processes

- Recent developments show that the same picture also holds under model uncertainty:

imprecise Markov process		**sublinear semigroup**		**generator**
$(X_t)_{t\geq 0}$	$\leftrightarrow$	$(\bar{S}_t)_{t\geq 0}$	$\leftrightarrow$	$\bar{A}$

- The solution of the Kolmogorov equation

$$\begin{cases} \partial_t u = \bar{A}u & \text{on } [0, T) \times \mathbb{R} \\ u(0, \cdot) = f & \text{on } \mathbb{R} \end{cases}$$

satisfies

$$\bar{E}\big[f(X_t) \mid X_0 = x\big] = \big(\bar{S}(t)f\big)(x) = u(t, x).$$

Here, $\bar{E}$ is an **upper expectation** or **sublinear expectation**.

Imprecise Markov processes

- Example: **G-Brownian motion** (S. Peng)
  - Starting with the solution of the *G-heat equation*

$$\begin{cases} \partial_t u = \sup_{\sigma \in [\underline{\sigma}, \overline{\sigma}]} \frac{\sigma^2}{2} \partial_{xx} u & \text{on } [0, T) \times \mathbb{R} \\ u(0, \cdot) = f & \text{on } \mathbb{R} \end{cases}$$

for imprecise volatilities in the interval $[\underline{\sigma}, \overline{\sigma}] \subset \mathbb{R}_+$,

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for imprecise volatilities in the interval $[\underline{\sigma}, \overline{\sigma}] \subset \mathbb{R}_+$, the *G-Brownian motion* $(X_t)_{t \geq 0}$ is the imprecise Markov process with upper transition probabilities

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  - The upper transition probabilities are *G-normally distributed* with mean x and imprecise variance $[t\underline{\sigma}^2, t\overline{\sigma}^2]$.

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  - The upper transition probabilities are *G-normally distributed* with mean x and imprecise variance $[t\underline{\sigma}^2, t\overline{\sigma}^2]$.
  - There exists an upper expectation $\bar{E}$ on the path space $C([0, \infty), \mathbb{R})$ with respective marginal distributions. $\rightsquigarrow$ *G-expectation*.

Semigroup envelope

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- Recall that the **fair price** of a contingent claim $f(X_T)$ with underlying price dynamics modelled by a transition semigroup $(S(t))_{t\geq 0}$ is given by

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- Example: Consider the family of generators $(A_\sigma)_{\sigma\in[\underline{\sigma},\overline{\sigma}]}$ with $A_\sigma f = \frac{\sigma^2}{2}f''$.

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- The goal is to compute "prices" under model uncertainty in a *cautious way*, i.e., we are looking for a family $\big(\bar{S}(t)\big)_{t\geq 0}$ such that, for all $f \in \mathrm{C_b}$,
  - (i) $\bar{S}(0)f = f$,
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  - (iii) $\sup_{\lambda\in\Lambda} S_\lambda(t)f \leq \bar{S}(t)f$ for all $t \geq 0$,
  - (iv) $\bar{S}(t)f \leq \bar{T}(t)f$ for all $t \geq 0$ and every family $\big(\bar{T}(t)\big)_{t\geq 0}$ satisfying (i) - (iii).

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  (iv) $\bar{S}(t)f \leq \bar{T}(t)f$ for all $t \geq 0$ and every family $\big(\bar{T}(t)\big)_{t\geq 0}$ satisfying (i) - (iii).

- In other words, $\big(\bar{S}(t)\big)_{t\geq 0}$ is the **semigroup envelope** (Nisio semigroup) of the family $(\bar{S}_\lambda)_{\lambda\in\Lambda}$.

Construction of the semigroup envelope

Let $(S_\lambda)_{\lambda\in\Lambda}$ be a **parameterized family of semigroups** $(S_\lambda(t))_{t\geq 0}$ on $\mathrm{C_b}$ such that

- $S_\lambda(t)\colon \mathrm{C_b}\to\mathrm{C_b}$ is linear and positive for all $t\geq 0$,
- $S_\lambda(0)f = f$ for all $f\in\mathrm{C_b}$,
- $S_\lambda(t+s)f = S_\lambda(t)(S_\lambda(s)f)$ for all $s,t\geq 0$ and $f\in\mathrm{C_b}$.

Construction of the semigroup envelope

Let $(S_\lambda)_{\lambda \in \Lambda}$ be a **parameterized family of semigroups** $(S_\lambda(t))_{t \geq 0}$ on $\mathrm{C_b}$.

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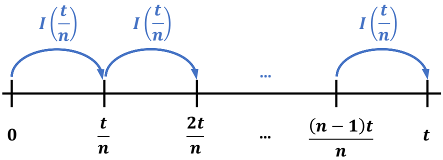

$\rightsquigarrow$ Consider the iterated operator

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3) Then, $\bar{S}(t)f := \lim_{n\to\infty} I(\frac{t}{n})^n f$ is the **semigroup envelope**.
4) Under reasonable assumptions, the generator $\bar{A}f := \lim_{h\downarrow 0}\frac{\bar{S}(h)f-f}{h}$ is given by

$$\bar{A}f = \sup_{\lambda\in\Lambda} A_\lambda f.$$

Semigroup envelope

- Example: **G-Brownian motion**
  - Let $(S_\sigma(t)f)(x) = E\big[f(x+\sigma B_t)\big]$ be the heat semigroup with volatility $\sigma \geq 0$.

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$$\big(I(t)f\big)(x) = \sup_{\sigma\in[\underline{\sigma},\overline{\sigma}]} E\big[f(x+\sigma B_t)\big]$$
describes the static upper transition probabilities with imprecise volatility.

    Note that $I(t)\colon C_b \to C_b$ satisfies for every $f, g \in \mathrm{C_b}$ and $c \in \mathbb{R}$,
    (i) $I(t)c = c$,
    (ii) $f \leq g$ implies $I(t)f \leq I(t)g$,
    (iii) $I(t)(f+g) \leq I(t)f + I(t)g$,

    i.e., the upper transition probability given by $f \mapsto (I(t)f)(x)$ is an *upper expectation* conditioned on the state x.

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describes the static upper transition probabilities with imprecise volatility.
  - The semigroup envelope
$$\bar{S}(t)f = \lim_{n\to\infty} I(\tfrac{t}{n})^n f$$
results in the transition semigroup of the G-Brownian motion with generator
$$\bar{A}f = \sup_{\sigma\in[\underline{\sigma},\overline{\sigma}]} \tfrac{\sigma^2}{2} f''.$$

Chernoff approximation

Chernoff-type approximations

- The construction of the semigroup envelope strongly relies on the fact that the approximation

$$I(\tfrac{t}{n})^n f \nearrow S(t)f \quad \text{is increasing in } n.$$

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- If $(I(t))_{t\geq 0}$ models the upper transition probabilities of a discrete-time imprecise Markov process $(X_{kt})_{k\in\mathbb{N}_0}$, the approximation is often not increasing. However, by relying on compactness arguments, under reasonable assumptions, one can still show that $\bar{S}(t)f := \lim_{n\to\infty} I(\frac{t}{n})^n f$ exists.

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  - **Random Walk** (with imprecise variance):

    $$\big(I(t)f\big)(x) = \bar{E}\big[f(x+\sqrt{t}\xi)\big] := \sup_{\sigma\in[\underline{\sigma},\overline{\sigma}]} E^{\sigma}\big[f(x+\sqrt{t}\xi)\big],$$

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    where $P^{\sigma}(\xi = \pm\sigma) = \frac{1}{2}$.

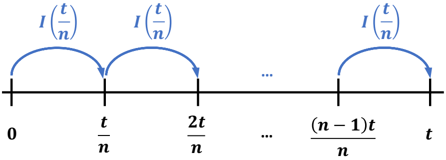
  - **Drift uncertainty:**

$$\big(I(t)f\big)(x) = \sup_{\mu} E\big[f(x+\mu(x)t+B_t)\big],$$

    where $B_t \sim N(0,t)$ and the supremum runs over a set of functions $\mu\colon \mathbb{R}\to\mathbb{R}$.

Chernoff-type approximations

- Let $(I(t))_{t\geq 0}$ be a family of one step operators $I(t)\colon \mathrm{C_b} \to \mathrm{C_b}$.
- Iteration for multiple time steps :



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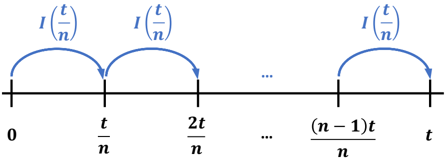

Define $I(\frac{t}{n})^n f := \underbrace{\left(I(\frac{t}{n}) \circ \cdots \circ I(\frac{t}{n})\right)}_{\text{n times}} f$.

- Pass to the continuous time limit $\bar{S}(t)f := \lim_{n\to\infty} I(\frac{t}{n})^n f$.
- Infinitesimal behaviour is given by

$$I'(0)f = \lim_{h\downarrow 0} \frac{I(h)f - f}{h} \quad \text{for all } f \in \mathrm{C}_\mathrm{b}^\infty.$$

Chernoff-type approximations

Assumption

Let $(I(t))_{t\geq 0}$ be a family of operators $I(t)\colon \mathrm{C_b} \to \mathrm{C_b}$ such that

(I1) $I(0)f = f$,

(I2) $I(t)$ is convex and monotone with $I(t)0 = 0$,

(I3) $\|I(t)f - I(t)g\|_\infty \leq e^{\omega t}\|f-g\|_\infty$,

(I4) $I(t)\colon \mathrm{Lip_b}(r) \to \mathrm{Lip_b}(e^{\omega t}r)$,

(I5) $\|I(t)(\tau_x f) - \tau_x I(t)f\|_\infty \leq Lrt|x|$ for $f \in \mathrm{Lip_b}(r)$,

(I6) $I'(0)f$ exists for $f \in \mathrm{C_b^\infty}$,

(I7) $(I(\frac{t}{n})^n)_{n\in\mathbb{N}}$ is uniformly continuous from above for $t \in [0, T]$.

Here, $\|\cdot\|_\infty$ denotes the supremum norm, $(\tau_x f)(y) := f(x+y)$ is the shifted function, $\mathrm{Lip_b}(r)$ is the set of all r-Lipschitz continuous functions, and $\mathrm{C_b^\infty}$ consists of all functions that have bounded derivatives of all orders.

Chernoff-type approximations

Theorem (Blessing and K.)

Let $(I(t))_{t\geq 0}$ be a family of operators $I(t)\colon \mathrm{C_b} \to \mathrm{C_b}$ satisfying (I1)–(I7). Then, there exists a strongly continuous convex monotone semigroup

$$\bar{S}(t)f := \lim_{n\to\infty} I(\tfrac{t}{n})^n f \quad \text{for all } t \geq 0 \text{ and } f \in \mathrm{C_b}.$$

Furthermore, the generator is given by

$$\bar{A}f = I'(0)f = \lim_{h\downarrow 0} \frac{I(h)f - f}{h} \quad \text{for all } f \in \mathrm{C_b^\infty}.$$

Let $(J(t))_{t\geq 0}$ be another family of operators satisfying (I1)–(I7) and denote by $(\bar{T}(t))_{t\geq 0}$ the corresponding semigroup. If $I'(0)f = J'(0)f$ for all $f \in \mathrm{C_b^\infty}$, then

$$\bar{S}(t)f = \bar{T}(t)f \quad \text{for all } t \geq 0 \text{ and } f \in \mathrm{C_b}.$$

Convex monotone semigroups

Definition

A strongly continuous convex monotone semigroup $(\bar{S}(t))_{t\geq 0}$ is a family of operators $\bar{S}(t)\colon \mathrm{C_b} \to \mathrm{C_b}$ satisfying the following conditions:

1. $\bar{S}(0)f = f$ and $\bar{S}(s+t)f = \bar{S}(s)\big(\bar{S}(t)f\big)$ for all $s, t \geq 0$ and $f \in \mathrm{C_b}$,
2. $\bar{S}(t)f \leq \bar{S}(t)g$ for all $t \geq 0$ and $f \leq g$,
3. $\bar{S}(t)(\lambda f + (1-\lambda)g) \leq \lambda \bar{S}(t)f + (1-\lambda)\bar{S}(t)g$ for all $t \geq 0$, $\lambda \in [0,1]$ and $f, g \in \mathrm{C_b}$,
4. The mapping $t \mapsto \bar{S}(t)f$ is continuous for all $f \in \mathrm{C_b}$.

The generator is defined by

$$D(\bar{A}) \to \mathrm{C_b},\ f \mapsto \lim_{h\downarrow 0} \frac{\bar{S}(h)f - f}{h}.$$

Random walk approximation

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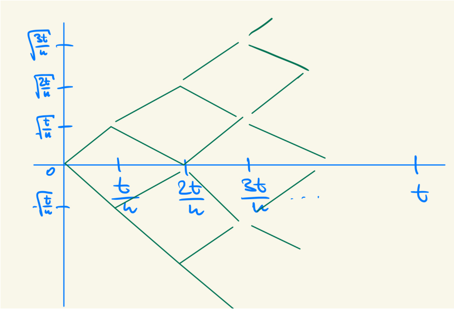

- Let $X_t^n = \sum_{i=1}^n \sqrt{\frac{t}{n}}\xi_i$, where $\xi_1, \xi_2, \ldots$ are iid with $P(\xi_k = \pm 1) = \frac{1}{2}$.

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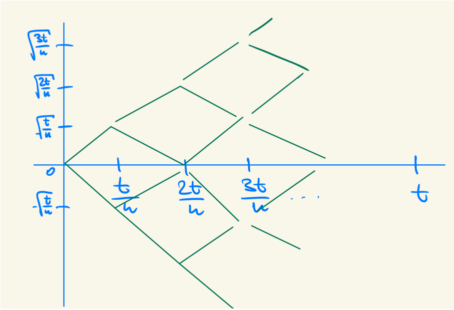

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- Then, it follows from the central limit theorem (CLT) that

$$\lim_{n\to\infty} E\big[f(X_t^n)\big] = E\big[f(B_t)\big] \quad \text{for all } f \in \mathrm{C_b},$$

where $B_t \sim N(0, t)$.

Random walk approximation

- Instead of relying on the CLT, we argue using the Chernoff approximation. To that end, we consider the transition probabilities of the random walk

$$\big(I(t)f\big)(x) = E\big[f(x+\sqrt{t}\xi_1\big]$$

and the transition probabilities of the Brownian motion (heat semigroup)

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- Both $(I(t))_{t\geq 0}$ and $(J(t))_{t\geq 0}$ satisfy (I1)–(I7). Moreover, for $f \in \mathrm{C}_\mathrm{b}^\infty$, it follows from Taylor's theorem that $I'(0) = J'(0) = \frac{1}{2}f''$.

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Indeed, since $f(x+\sqrt{t}\xi_1) = f(x) + f'(x)\sqrt{t}\xi_1 + \frac{1}{2}f''(x)t\xi_1^2 + o(t)$, we obtain

$$\left(\frac{I(t)f-f}{t}\right)(x) = \frac{E\left[\frac{1}{2}f''(x)t\xi_1^2\right] + o(t)}{t} \to \frac{1}{2}f''(x) \quad \text{for } t\downarrow 0.$$

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- Both $(I(t))_{t\geq 0}$ and $(J(t))_{t\geq 0}$ satisfy (I1)–(I7). Moreover, for $f \in \mathrm{C}_\mathrm{b}^\infty$, it follows from Taylor's theorem that $I'(0) = J'(0) = \frac{1}{2}f''$.
- Hence, we can apply the previous theorem and obtain for every $f \in \mathrm{C}_\mathrm{b}$,

$$\Big(I\big(\tfrac{t}{n}\big)^n f\Big)(0) = E\bigg[f\Big(\sum_{i=1}^n \sqrt{\tfrac{t}{n}}\xi_i\Big)\bigg] \longrightarrow \big(J(t)f\big)(0) = E\big[f(B_t)\big] \quad \text{for } n\to\infty,$$

(note that $J(\frac{t}{n})^n f = J(t)f$ because $(J(t))_{t\geq 0}$ is a semigroup).

Random walk approximation

- Likewise, for the **random walk with imprecise variance** with upper transition probabilities

$$\big(I(t)f\big)(x) = \bar{E}\big[f(x+\sqrt{t}\xi_1)\big] := \sup_{\sigma\in[\underline{\sigma},\overline{\sigma}]} E^\sigma\big[f(x+\sqrt{t}\xi_1)\big],$$

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Here, $(\bar{S}(t))_{t\geq 0}$ is the transition semigroup of the G-Brownian motion.

- In particular, the distribution of

$$\sum_{i=1}^{n}\sqrt{\tfrac{t}{n}}\xi_i$$

converges to the G-normal distribution with mean 0 and imprecise variance $[t\underline{\sigma}^2, t\overline{\sigma}^2]$. This is a version of the sublinear CLT of S. Peng.

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Thank you